

# Software demo: HoPeS

## Cloud segmentation and skeletons

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is supported by the EPSRC  
*Impact Acceleration Account*

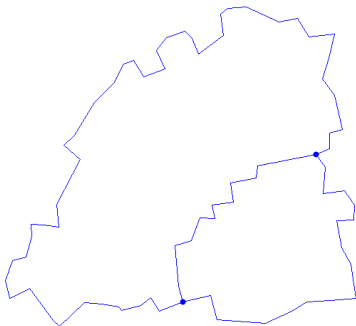


# 2D cloud software: HoPeS

**Input** :  $n$  points  $C \subset \mathbb{R}^2$  with real coordinates

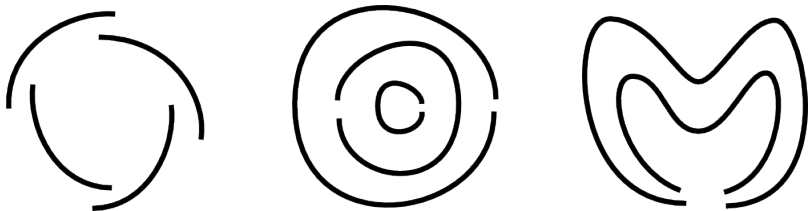
**Time**: guaranteed  $O(n \log n)$  in the worst case

**Output** : persistent hole boundaries, skeletons



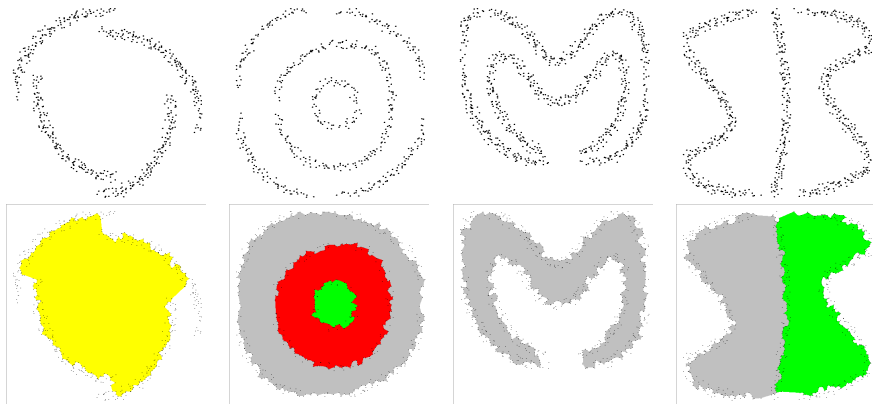
# Computer Graphics application

**Problem:** complete all closed contours or paint all regions that they enclose (a *segmentation*).



A user drawing a sketch on a tablet might be happy with our fast automatic ‘best guess’:  
*make contours closed* so that I can paint areas  
(a scale is easy to find, but we can’t ask for it).

# Cloud segmentation into regions

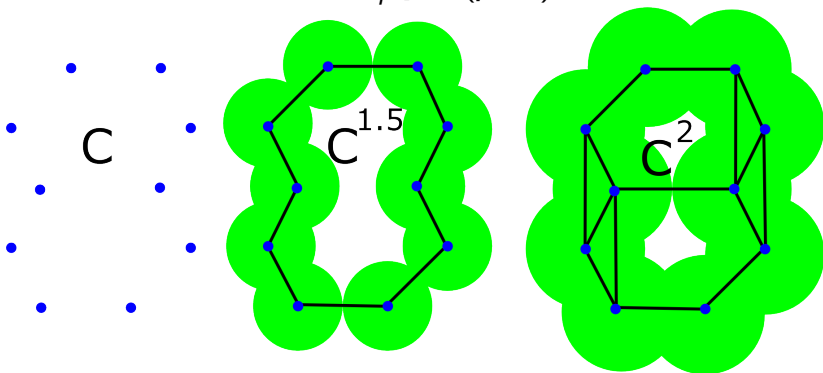


Proved: contours are close to the ground truth.

VK, Pattern Recognition Letters, to appear in 2016

# From a cloud to a filtration

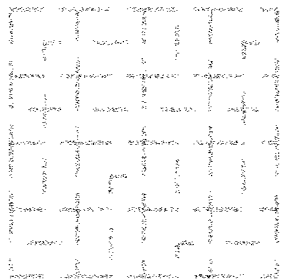
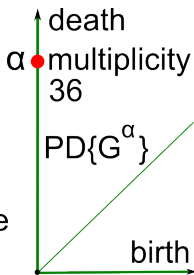
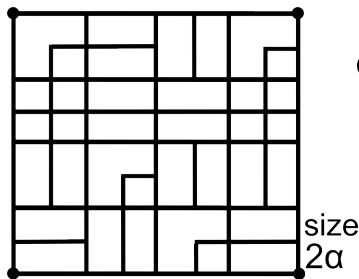
**Def :** the  $\alpha$ -offset of a cloud  $C \subset \mathbb{R}^2$  is the union of closed balls  $C^\alpha = \cup_{p \in C} B(p; \alpha)$  of a radius  $\alpha$ .



**Filtration**  $C = C^0 \subset \dots \subset C^\alpha \subset \dots \subset C^{+\infty} = \mathbb{R}^2$ .

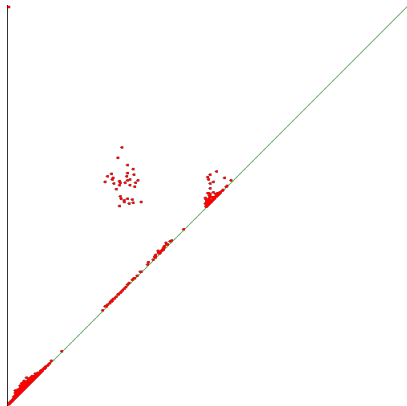
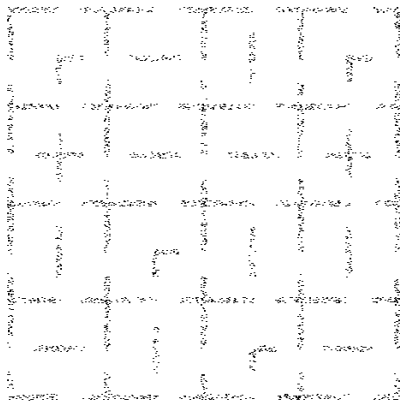
# Counting holes in $C$ may be easy

The graph  $G$  has  $H_1$  of rank 36, hence any  $\varepsilon$ -sample  $C$  of  $G$  will probably have 36 holes.



How can we see that there are 36 holes in  $C$ ?

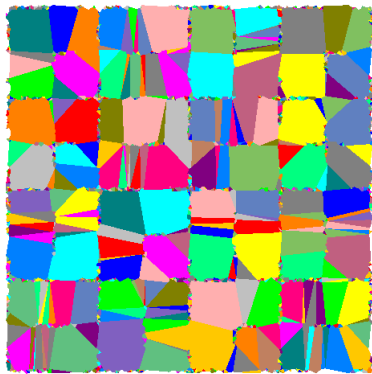
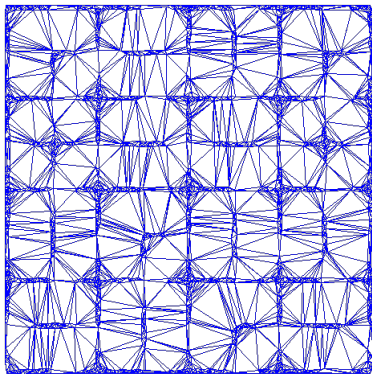
# Using stability of persistence



We can find the *widest diagonal gap* separating 36 points from the rest of persistence diagram.

# An initial segmentation of $C$

Acute Delaunay triangle is a 'center of gravity'.

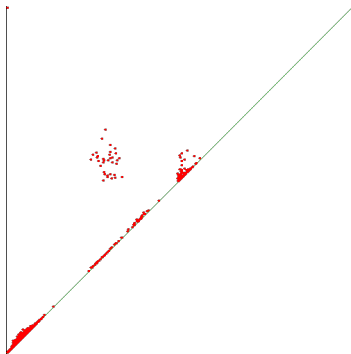
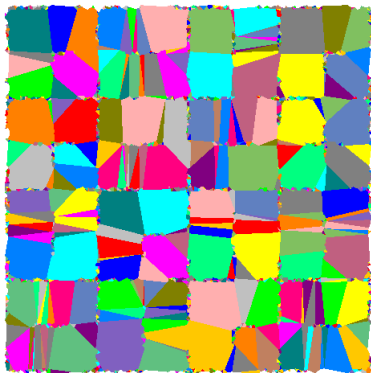


We attach all adjacent non-acute triangles to get an initial segmentation on the right hand side.



# Harder than counting cycles

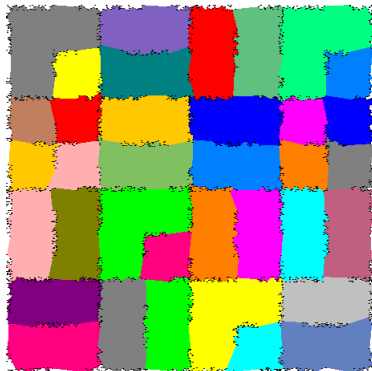
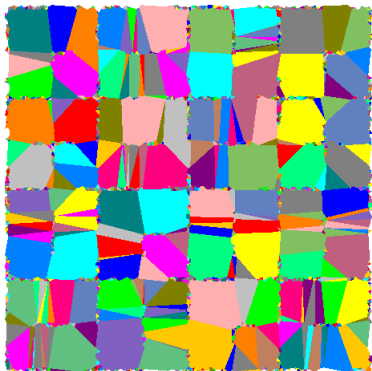
Initial regions  $\leftrightarrow$  red dots in PD (too many).



We should merge 36 regions of high persistence with all remaining regions of lower persistence.

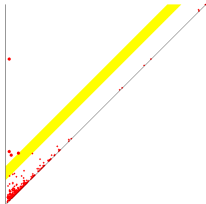
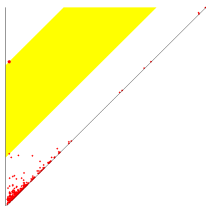
# Merging initial regions

Building  $PD\{C^\alpha\}$ , we keep adjacency relations of merged regions to enrich persistence info.



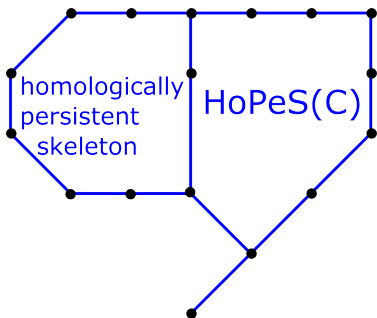
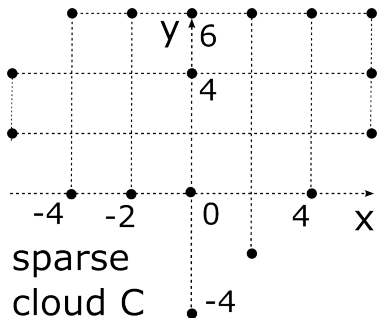
# Hierarchy of segmentations

A user can choose to get exactly  $k$  regions by choosing 2nd widest diagonal gap in PD1 etc.



# Parameterless skeletonisation

**Def :** **Homologically Persistent Skeleton** of a cloud  $C$  is  $\text{HoPeS}(C) = \text{MST}(C) \cup \text{critical edges}$  representing all dots in 1D persistence of  $\{C^\alpha\}$ .



# Properties of $\text{HoPeS}(C)$

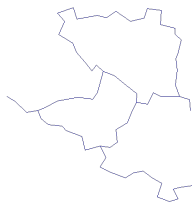
**Optimality** : for any scale  $\alpha$ , reduced subgraph  $\text{HoPeS}(C; \alpha)$  is *shortest* among all graphs  $G \subset C^\alpha$  inducing isomorphisms in  $H_0, H_1$ .

**Reconstruction** : if  $C$  is an  $\varepsilon$ -sample of a good  $G$ , derived  $\text{HoPeS}_{k,l}(C) \sim G$  are  $2\varepsilon$ -close to  $G$ .

**Global stability** :  $\text{HoPeS}(C)$  remains in a small offset *after perturbing*  $C$ . Proofs and extension: VK, Computer Graphics Forum 34-5 (2015), presented at SGP 2015: Symposium on Geometry Processing.

# Recognising visual markers

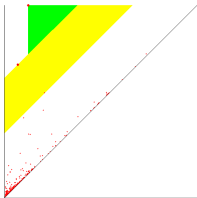
Shop barcodes are not readable by humans.



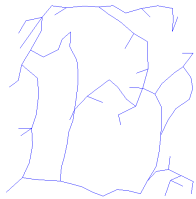
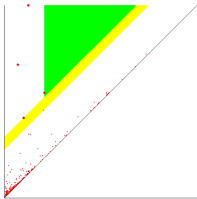
We can make *visual markers* like Egyptian hieroglyphs **readable** by *humans and robots*.

VK, CAIP'15: Computer Analysis of Images and Patterns

# Fast simplification of images



1st widest gap gives contours of 2 large peppers



2nd widest gap gives 2 more small peppers.

# Summary: C++ code HoPeS

- *time*  $O(n \log n)$  for any input cloud  $C \subset \mathbb{R}^2$
- *persistent structures* directly on data  
with guarantees: boundary contours,  
Homologically Persistent Skeleton HoPeS
- first persistence software *in England*

Papers and C++ code are at <http://kurlin.org>.

*Collaborations and applications* are welcome!